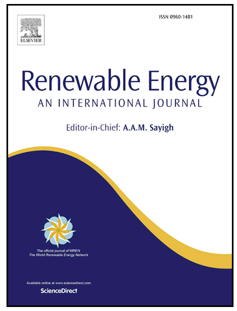

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Abstract

Wind energy is one of the most promising option for the renewable energy. Finding optimum set of locations for wind turbines in a wind farm so that the total energy output of the farm is maximum, is usually referred as the wind farm layout optimization problem (WFLOP). This article presents the solution of WFLOP using a recent unconventional optimization algorithm, Biogeography Based Optimization (BBO). In this article, for a given wind farm not only the optimum locations of the wind turbines are obtained but also the maximum number of turbines is recommended. Experiments have been carried out for wind farms of various sizes. BBO has shown to outperform as compare to earlier methodologies of solving WFLOP.

Keywords: Wind energy, Renewable energy, Wind farm, Wind turbines, Biogeography-based optimization

1. Introduction

Wind energy is the most precious gift of nature to the world. The advance technology is trying to find out alternative of nonrenewable energy resources using wind energy. Now advance technology is developed to generate electricity from wind energy. Now a days, conventional windmills have been substituted by specially designed wind turbines for increasing the production of electricity. Wind turbine converts the wind energy into electricity.

Wind farm layout optimization (WFLO) is the pattern of wind turbines scheme subject to the constraints related to the position of the turbines, rotor radius and farm radius. In the wind farm layout optimization problem (WFLOP) model, the objective function is the maximization of expected power. The solution of this problem is to find the optimal placement of wind turbines so that the expected energy output of the whole wind farm is maximum. The complexity of WFLOP model depends on the constraints type.

The wake model depends on the thrust and the turbulence level at the turbine. The wake from one turbine will be detrimental on the wind speed and turbulence at down wind turbines. The effects of the wake spread out downwind and decay with distance according to generalized wake models. The effect of the wake is measured in the specific range. If the turbines are located within the range of four rotor diameter, they get affected by wake.

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Significant development has been taken place in the machinery of wind energy production. The percentage of wind energy production is increasing rapidly. In near future also the wind energy production is expected to increase. The inherent challenge with wind energy is its product cost. This challenge can be controlled by the optimal wind farm layout design. Wind farm layout optimization problem is being solved from many years. Researchers are continuously developing the new approaches of designing and solving WFLOP. Lackner et al. [1] provided an analytical framework for offshore wind farm layout optimization. Here the annual energy production of the wind farm is fully dependent on the turbines position. Castro et al. [2] presented a genetic algorithm for the optimal design of the wind farms. In [3], Elkinton et al. presented offshore wind farm layout using several optimization algorithms. There are limited efforts done by optimization community to solving WFLOP. Mosetti et al. [4] and Grady et al. [5] demonstrated the placement of wind turbines using binary coded GA (genetic algorithm) for maximizing energy production. Haung et al. [6] applied the distributed GA to finding more effective optimal solution of WFLOP. Emami et al. [7] introduced a new approach on optimal placement of wind turbines using GA with additional property, the controlling capability of wind farm construction cost in objective function. Şişbot et al. [8] used a multi-objective GA to solving WFLOP. M. Samorani [9] demonstrated WFLOP consting of two conflicting problem as maximization of expected power production with minimization of wake effect within several turbines. Ozturk et al. [10] developed greedy heuristic methodology for wind energy conversion system positioning. Bilbao et al. [11] applied simulated annealing (SA) to compute the optimal placement of wind turbines in a wind farm to produce maximum power. Rivas et al. [12] also applied the simulated annealing algorithm to solve wind turbine positioning problem. Kusiak et al. [13] presented a generic model for wind farm layout optimization based on wind distribution. In [13], evolutionary strategy is considered for optimizing layout up to 6 number of turbines in the circular wind farm. Wagner et al. [14] presented a better evolution strategy, named as covariance matrix adaptation based evolutionary strategy (CMA-ES) for maximum power production. Yeng yin et al. [15] developed a combined algorithm named as greedy randomized adaptive search procedures algorithm with variable neighborhood search algorithm (GRASP-VNS) for optimal placement of wind turbines. In [16] and [17], Eroğlu et al. developed ant colony optimization (ACO) and particle filtering (PF) approach to solve WFLOP, respectively. These intensive uses of metaheuristic algorithms to solve WFLOP inspire researchers to explore other recent metaheuristics also for the same.

This article presents relatively a recent approach Biogeography-based optimization algorithm (BBO) to solving WFLOP. The main objective of this article is to investigate the applicability of the BBO algorithm in solving WFLOP. In this article, we try to find out the optimal locations of wind turbines and maximum possible number of wind turbines in the wind farms with radii 500 (m), 750 (m) and 1000 (m).

Rest of the article is organized as follows. The problem modeling and statement are described in section 2. Section 3 details of BBO algorithm. In section 4, BBO algorithm is applied to solve WFLOP model. In section 5, various numerical experiments, comparison of results and discussions are given. The article concluded in section 6.

2. Problem modeling and statement

Some basic definitions are required to constructing the wind farm and to finding the optimal placement of turbines. It is important to make some assumptions to solving the WFLOP.

1. The number of turbines N is fixed before the planning of the wind farm construction because investment in the wind farm project depends on the number of turbines. For example, a 30 MW wind farm project, requires 20 number of wind turbines of capacity 1.5 MW each.
2. Location of each turbine in the farm is represented in the form of two-dimensional co-ordinates (x, y) and length of the location vector of each turbine is given by $\sqrt{x^2 + y^2}$. Here only slight changes in surface roughness and the optimal solution of WFLOP is represented by the N positions (x_i, y_i), i=1,......,N for N number of turbines.
3. All turbines in the wind farm are considered to be uniform with respect to both external quality (design, brand, model, hub height) and internal quality (power curve, theoretical power, capacity).
4. For a given location, height and direction, wind speed v follows a Weibull distribution $p_v(v, k, c) = \frac{k}{c}(\frac{v}{c})^{(k-1)}e^{-(\frac{v}{c})^k}$, where k is the shape parameter, c is the scale parameter and $p_v(.)$ is the probability density function. This assumption is very common for many windy sites [18].
5. One of the parameter of Weibull distribution function is wind speed v which is a function of wind direction θ then $v = v(\theta)$, i.e. $k = k(\theta)$, $c = c(\theta)$, $0^0 \leq \theta \leq 360^0$. Thus, the wind direction θ is a significant parameter of WFLOP. Fig.1 gives the pictorial description of wind direction for proposed work, where θ =0^0, 90^0, 180^0 and 270^0 represents east, north, west and south, respectively.
6. There must be a proper space between two turbines. Proper spacing between turbines reduces some dangerous loads on turbines, e.g. wind turbulence. If $T_i(x_i, y_i)$ and $T_j(x_j, y_j)$ be two turbines then they should satisfy the inequality $(x_i - x_j)^2 + (y_i - y_j)^2 \geq 64R^2$, where R is the given rotor radius.
7. WFLOP is a layout optimization problem. Thus the primary task of this study is to consider the wind farm layout boundary. We can take elliptical, circular or any other shape of the wind farm. We have selected circular shape of the wind farm as a boundary for this study.
8. All the turbines must be situated within the farm. Thus any turbine T_i with Cartesian coordinate (x_i, y_i) must satisfy the constraint $x_i^2 + y_i^2 \leq r^2$, where r is the radius of the wind farm. In this study, wind farms of radii 500 (m), 750 (m) and 1000 (m) are considered.
9. Search space of the problem is bounded by the wind farm shape and has continuous coordinate variables. Therefore, to locate a wind turbine, the grid system is not required.
10. Mathematical model of the problem consists of two parts: wake effect and power output model. Wake effect causes lower power generation of downstream turbines. The Jensen's wake model [19, 20] is used and adopted the continuous search space of WFLOP. Power output model is considered from Kusiak et al. [13].

11. The objective of this study is to maximize power output in such a way that wake effect model can be minimized with two constraints obtained from assumptions 6 and 8, i.e., the spacing between any two turbines is at least four rotor diameters and all turbines must be situated within the farm.

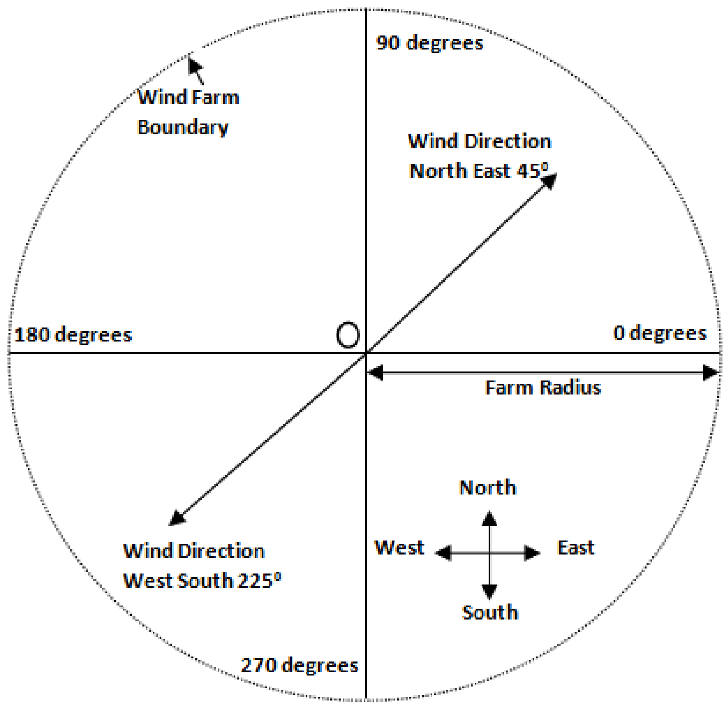


Figure 1: A typical circular wind farm with wind directions [13]

2.1. The wake effect model

Wake loss is a vital component in the wind farm layout design [21]. When a uniform wind encounters a turbine, behind the turbine, a linearly expanding wake appears [19, 1]. Because of this, a part of the wind's speed will be reduced from it's original speed v_{up} to v_{down}. Fig. 2 gives the pictorial description of the basic concept of the wake behind a wind turbine. Here v_{up} indicates the actual wind speed and v_{down} indicates the wind speed after wake, K indicates the wake spreading constant and d indicates the distance between two turbines.

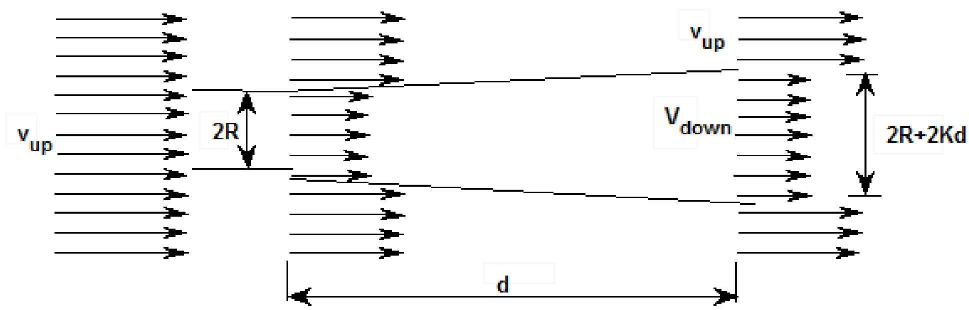


Figure 2: Wake model of wind turbine [13]

The velocity deficit is given by the following equation.

$$Vel\_def_{ij} = 1 - \frac{v_{down}}{v_{up}} = \frac{1-\sqrt{1-c_T}}{(1+Kd_{ij}/R)^2} \tag{1}$$

Where vel$_def_{ij}$ indicates the velocity deficit at turbine i due to the wake of turbine j, c_T represents the thrust coefficient of the turbine and d_{ij} indicates the distance between turbine i and turbine j, projected on wind direction θ.
In the case of a turbine is affected by wakes of more than one turbine. The overall velocity deficit for that turbine is calculated by the following equation.

$$Vel\_def_i = \sqrt{\sum_{j\ =\ 1, j\neq\ i}^{N} vel\_def_{ij}^2} \tag{2}$$

Where vel_def_i indicates the total wind speed deficit at turbine i.
Given wind direction θ, all turbines' rotors are normally positioned perpendicular to the wind direction. The wake behind turbine could be seen as a part of an imaginary cone. Fig. 3, represents a half cone formed by a turbine located at (x, y). Here A is the imaginary vertex. Parameter $\alpha(0 \leq \alpha \leq \pi/2)$ is evaluated as $\arctan(K)$.

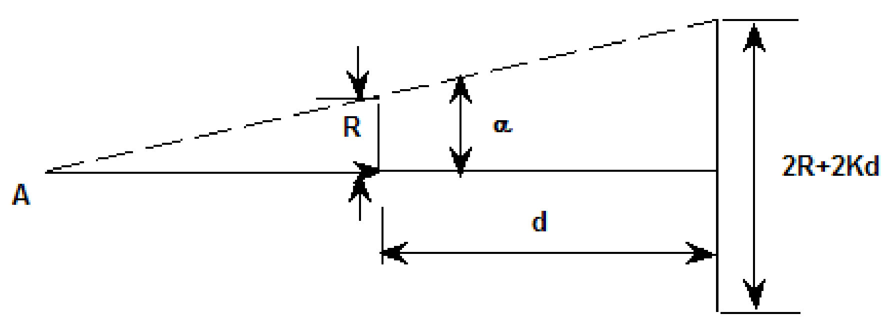


Figure 3: An imaginary half cone of a wind turbine [13]

Lemma 1. For the given wind direction θ, the angle β_{ij}, $0 \leq \beta_{ij} \leq \pi$, between vector, originating at A to turbine i and the vector, originating at A to turbine j, β_{ij} is computed as

$$\beta_{ij} = cos^{-1}\left\{ \frac{(x_i - x_j)cos\theta + (y_i - y_j)sin\theta + \frac{R}{K}}{\sqrt{(x_i - x_j + \frac{R}{K}cos\theta)^2 + (y_i - y_j + \frac{R}{K}sin\theta)^2}} \right\} \tag{3}$$

Where R/K indicates the distance between rotor center and A.

Lemma 2. If wind turbine i is under the wake effect of turbine j, distance between turbine i and turbine j projected on the wind direction θ is,

$$d_{ij} = \mid (x_i - x_j)cos\theta + (y_i - y_j)sin\theta \mid$$

Equation (2) can be written as

$$Vel\_def_i = \sqrt{\sum_{j=1,\ j\neq i,\ \beta_{ij}<\alpha}^{N} vel\_def_{ij}^2} \tag{4}$$

Equation (4) expresses that vel_def_i is a function of wind direction θ and location of turbines (x_i, y_i).

Only scaling parameter c of Weibull distribution is affected by wake loss and is given by equation (5) [1].

$$c_i(\theta) = c(\theta) \times (1 - vel\_def_i), i = 1, ..., N \tag{5}$$

Where $c_i(\theta)$ is some function of θ for a given turbine i.

2.2. The power model

The power curve model is demonstrated by following.

$$f(v) = \begin{cases} 0, & v < v_{cut-in} \\ \lambda' v + \eta, & v_{cut-in} \leq v \leq v_{rated} \\ P_{rated}, & v_{cut-out} > v > v_{rated} \end{cases} \tag{6}$$

Where v_{cut-in} is the cut-in wind speed. There is no power output if the wind speed is less than v_{cut-in} because of low torque. The power output is static, i.e.,P_{rated} if the wind speed is between rated speed and cut-out speed. Power output is represented by linear form between cut-in wind speed and rated wind speed. λ' expresses the slope parameter and η expresses as intercept parameter.

The expected energy output of single turbine located at (x, y) and wind direction θ is expressed as follows

$$\begin{aligned} E(P_i) &= \int_0^{360} p_\theta(\theta) E(P_i, \theta) d\theta \\ &= \int_0^{360} p_\theta d\theta \times \int_0^{\infty} f(v) p_v(v, k(\theta), c_i(\theta)) dv \end{aligned} \tag{7}$$

The objective function is to maximize the total energy production of the wind farm subject to assumptions (6) and (8). The optimization problem expressed by mathematical model:

$$
\begin{aligned}
& max \sum_{i=1}^{N} E(P_i) \\
& \text{s.t.} \\
& (x_i - x_j)^2 + (y_i - y_j)^2 \geq 64R^2, i, j = 1, 2, ..., N, i \neq j \\
& x_i^2 + y_i^2 \leq r^2
\end{aligned} \tag{8}
$$

Where $E(P_i)$ represents the power output of the i^{th} turbine. Though, $P{=}f(v)$ simulates a sigmoid function, it can be mathematically approximated as a linear function with tolerable error.

Equation (7) can be written using equation (6) and assumption (4) as follows:

$$
\begin{aligned}
E(P_i) &= \int_0^{360} p_\theta d\theta \times \int_0^{\infty} f(v) p_v(v, k(\theta), c_i(\theta)) dv \\
&= \int_0^{360} p_\theta d\theta \times \int_0^{\infty} f(v) \frac{k(\theta)}{c_i(\theta)} \left(\frac{v}{c_i(\theta)} \right)^{k(\theta)-1} e^{-(v/c_i(\theta))^{k(\theta)}} dv \\
&= \int_0^{360} p_\theta d\theta \times \left(\lambda \int_{v_{cut-in}}^{v_{rated}} v \frac{k(\theta)}{c_i(\theta)} \left(\frac{v}{c_i(\theta)} \right)^{k(\theta)-1} e^{-(v/c_i(\theta))^{k(\theta)}} dv \right. \\
&+ \eta \int_{v_{cut-in}}^{v_{rated}} \frac{k(\theta)}{c_i(\theta)} \left(\frac{v}{c_i(\theta)} \right)^{k(\theta)-1} e^{-(v/c_i(\theta))^{k(\theta)}} dv \\
&+ \left. P_{rated} \int_{v_{rated}}^{\infty} \frac{k(\theta)}{c_i(\theta)} \left(\frac{v}{c_i(\theta)} \right)^{k(\theta)-1} e^{-(v/c_i(\theta))^{k(\theta)}} dv \right)
\end{aligned} \tag{9}
$$

Wind direction is discretized into small bins so that the integration part can be approximated with the Riemann sum [22]. Let wind direction is discretized into $N_\theta + 1$ bins of equal width. All discretized part of wind directions between 0^0 to 360^0 are $\theta_0 = 0^0$, θ_1, θ_2,....., θ_{N_θ}, $\theta_{N_\theta+1} = 360^0$. Wind speed is also discretized into $N_v + 1$ bins of equal width. All discretized part of wind speed between v_{cut-in} and v_{rated} are $v_0 = v_{cut-in}$, v_1, v_2,....., v_{N_v}, $\theta_{N_v+1} = v_{rated}$. Finally the expected energy output of i^{th} turbine is transformed in discretized form and is given below. Detail description about this discretization of expected energy output of single turbine can be found in [13].

$$
\begin{aligned}
E(P_i) &= \lambda' \sum_{j=1}^{N_v+1} \left(\frac{v_{j-1}+v_j}{2} \right) \sum_{l=1}^{N_\theta+1} \Bigg\{ (\theta_l - \theta_{l-1})\omega_{l-1} \\
&\quad \Bigg\{ e^{-\left(v_{j-1}/c_i\left(\frac{\theta_l+\theta_{l-1}}{2}\right)\right)^{k\left(\frac{\theta_l+\theta_{l-1}}{2}\right)}} - e^{-\left(v_j/c_i\left(\frac{\theta_l+\theta_{l-1}}{2}\right)\right)^{k\left(\frac{\theta_l+\theta_{l-1}}{2}\right)}} \Bigg\} \Bigg\} \\
&+ P_{rated} \sum_{l=1}^{N_\theta+1} (\theta_l - \theta_{l-1})\omega_{l-1} e^{-\left(v_{rated}/c_i\left(\frac{\theta_l+\theta_{l-1}}{2}\right)\right)^{k\left(\frac{\theta_l+\theta_{l-1}}{2}\right)}} \\
&+ \eta \sum_{l=1}^{N_\theta+1} \Bigg\{ (\theta_l - \theta_{l-1})\omega_{l-1} \Bigg\{ e^{-\left(v_{cut-in}/c_i\left(\frac{\theta_l+\theta_{l-1}}{2}\right)\right)^{k\left(\frac{\theta_l+\theta_{l-1}}{2}\right)}} \\
&- e^{-\left(v_{rated}/c_i\left(\frac{\theta_l+\theta_{l-1}}{2}\right)\right)^{k\left(\frac{\theta_l+\theta_{l-1}}{2}\right)}} \Bigg\} \Bigg\}
\end{aligned} \tag{10}
$$

Where N_v expresses the number of the intervals for wind speed. N_θ expresses the number of the intervals for wind direction. ω_{l-1} is the blowing probability of the $(l-1)^{th}$ wind direction interval. The resultant optimization problem is a complex, nonlinear, constrained optimization problem. Therefore, modern derivative-free optimization algorithm becomes important to solve the model. This article uses BBO to solve the model (8) with (10).

3. Biogeography Based Optimization

The popular method of studying geographical distribution of biological organisms is biogeography, and whose earliest works can be traced back to the days by Alfred Wallace and Charles Darwin [23]. The mathematical model of biogeography has come in the picture due to Robert Mac Arther and Edward Wilson, which describes the migration of species from one island to another island, the arrival of new species and the extinction of some existing species [24]. Recently a new evolutionary population-based optimization technique has been proposed occupying the basic nature of biogeography and is named as biogeography-based optimization (BBO) [23]. However, the study of biogeography contains evolution, migration and extinction but BBO is inspired by only migration of species among islands. In biogeography model, the fitness of a geographical area is assessed on the basis of habitat suitability index (called HSI). Habitats which are more suitable for species to reside are said to have high HSI. Similarly, habitats which are less suitable for species to reside are said to have low HSI. In this way high HSI habitats have the relatively larger number of species. The characterization of habitability is called suitability index variable ($SIVs$) for example rainfall, vegetation, temperature, etc. The migration of species among different habitats is mainly controlled by two parameters, immigration rate (λ) and emigration rate (μ). λ and μ are the functions of the number of species in a habitat. $P_s(t)$ is the probability that there are s species in the habitat at any time t.

$$P_s(t+\Delta t) = P_s(t)(1-\lambda_s\Delta t - \mu_s\Delta t) + P_{s-1}\lambda_{s-1}\Delta t + P_{s+1}\mu_{s+1}\Delta t \tag{11}$$

Where λ_s is immigration rate when there are s species in the habitat. μ_s is emigration rate when there are s species in the habitat.
At time t+Δt one of the following condition must hold for s species in the habitat:

1. If there were s species in the habitat at time t. Then there will be no immigration and no emigration of species within time t and t+Δt.
2. If there were $(s-1)$ species in the habitat at time t. Then one species immigrate between time t and t+Δt.
3. If there were $(s+1)$ species in the habitat at time t. Then one species emigrate between time t and t+Δt.

For ignoring the probability of more than one immigration or emigration during Δt , we take Δt very small.
Taking $\Delta t \longrightarrow 0$

$$\dot{P}_s = \begin{cases} -(\lambda_s + \mu_s)P_s + \mu_{s+1}P_{s+1}, & s = 0 \\ -(\lambda_s + \mu_s)P_s + \lambda_{s-1}P_{s-1} + \mu_{s+1}P_{s+1}, & 1 \leq s \leq s_{max} - 1 \\ -(\lambda_s + \mu_s)P_s + \lambda_{s-1}P_{s-1}, & s = s_{max} \end{cases} \tag{12}$$

We can obtain a matrix relation executing the dynamic equations of the probabilities of the number of species in the habitat.

$$\begin{bmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \vdots \\ \vdots \\ \dot{P}_{S_{max}} \end{bmatrix} = \begin{bmatrix} -(\lambda_0 + \mu_0) & \mu_1 & 0 & \cdots & 0 \\ \lambda_0 & -(\lambda_1 + \mu_1) & \mu_2 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \lambda_{n-2} & -(\lambda_{n-1} + \mu_{n-1}) & \mu_n \\ 0 & \cdots & 0 & \lambda_{n-1} & -(\lambda_n + \mu_n) \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ \vdots \\ \vdots \\ P_{S_{max}} \end{bmatrix} \tag{13}$$

The primary concept of biogeography has been used to design a population-based optimization procedure that can be potentially applied to optimize many engineering optimization problems. BBO is based on the two simple biogeography concepts, migration and mutation. In BBO, each habitat H represents a potential solution vector of $m \times 1$; Where m is the number of $SIVs$ of the habitat. We find out HSI of each habitat which corresponds to fitness function of population-based algorithms. Habitat with highest HSI reveals that it is the best candidate for the optimum solution among all habitats. It is considered that the ecosystem constitutes N_p habitats i.e. the population size is N_p. In the basic BBO algorithm, the immigration and emigration rates linearly vary with number of species, as shown in Fig. 4 and they can be calculated by the following formulae:

$$\lambda_j = I\left(1 - \frac{k_j}{n}\right) \tag{14}$$

$$\mu_j = E\left(\frac{k_j}{n}\right) \tag{15}$$

λ_j stands for immigration rate of j^{th} individual (island).
μ_j stands for emigration rate of j^{th} individual (island).

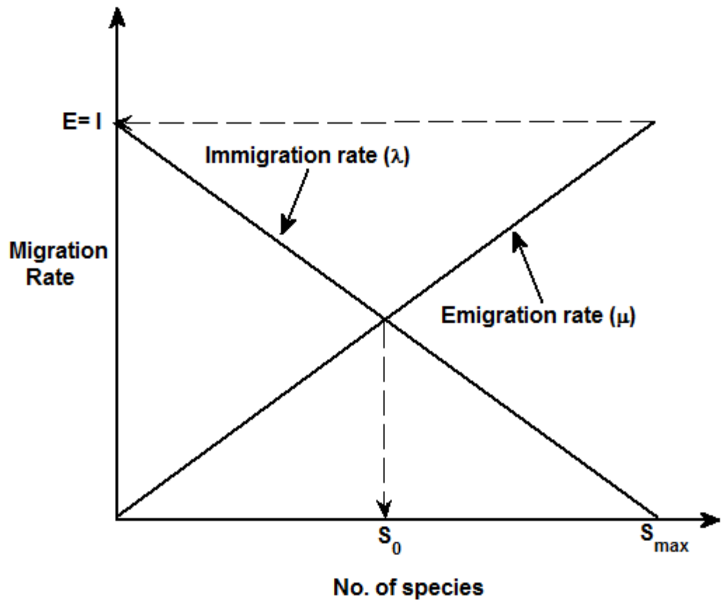


Figure 4: Relation between number of species and migration rate [23]

I stands for maximum possible immigration rate.
E stands for maximum possible emigration rate.
n stands for maximum possible number of species that island can support.
k_j stands for fitness rank of j^{th} island after sorting of fitness, so that for worst solution k_j is taken as 1 and for best solution k_j is taken as n.
It is sufficient to assume a linear relationship between the number of species and migration rate for many application points of views. The relation between migration rate (λ and μ) and the number of species are illustrated in Fig. 4. If there are no species in the island then immigration rate is maximum, denoted by I. If there are the maximum number of species (S_{max}) in the island then emigration rate is maximum, denoted by E. At the state of an equilibrium number of species denoted by S_0 and in an equilibrium state, immigration rate and emigration rate are equal. The islands referred as high HSI islands if the number of species is more than S_0 and the islands referred as low HSI islands if the number of species is less than S_0. Algorithm 1 describes the pseudo code of BBO.

Algorithm 1 Biogeography-based optimization algorithm

```
Initialize the population
Population size ← N_p;
Sort the population according to the increasing order of fitness
Calculate λ and μ
Generation index ← GenIndex;
for GenIndex = 1 to MaxGen do
  Apply migration
  for j = 1 to N_p do
    Select habitat H_j according to λ_j
    if rand(0,1) < λ_j then
      for e = 1 to N_p do
        Select habitat H_e according to μ_e
        Replace the selected SIV of H_j by randomly selected SIV of H_e
      end for
    end if
  end for
  Apply mutation
  for j = 1 to N_p do
    Compute mutation probability m(S)
    if rand(0,1) < m(S) then
      Replace H_j(SIV) with randomly generated SIV
    end if
  end for
  Sort the population according to the increasing order of fitness
  Keep the elite solution
  Stop, if termination criterion satisfied
end for
```

Migration and mutation are two crucial operators in BBO. "Migration" and the "Mutation" procedures are responsible to evolve new candidate solutions. This procedure of governing the habitats to the "Migration" procedure, followed by the "Mutation" procedure, is continued to next generation until the termination criterion is reached. This criterion can be the maximum number of generations or obtaining the desired solution. The basic concept of migration procedure is the probabilistically share the information between habitats by utilizing the immigration rate (λ_s) and emigration rate (μ_s). The migration operator is same as the crossover operator of the evolutionary algorithms and is responsible for sharing the features among candidate solutions for modifying fitness. In the migration procedure, immigrating habitat is selected according to the probability of immigration rate and emigrating habitat is selected according to the probability of emigration rate. Then probabilistically decide which of the SIV of immigrating habitat is required to be modified. Once an SIV is selected, replace that SIV by emigrating habitat's SIV. The other important phenomenon is mutation. Mutation occurs by sudden changes in islands due to the random event and is responsible for maintaining the diversity of island in BBO process. Analysis of Fig. 4 reveals that very high HSI

solutions and very low HSI solutions have very low probability while medium HSI solutions have the relatively high probability to exist as a solution. So mutation approach gives the same chance to improve low HSI solutions as well as high HSI solutions. The mutation rate $mut(j)$ can be expressed as:

$$mut(j) = m_{max}\left(1 - \frac{P_j}{P_{max}}\right) \tag{16}$$

Where m_{max} is the user defined parameter and $P_{max} = max\{P_j\}$; j=1, 2,...., N_p.

4. Biogeography-based optimization for wind farm layout optimization problem

In the literature, wind farm layout optimization problem (WFLOP) has been dealt with many nature inspired optimization algorithms, e.g. GA, PSO, PF, ACO, CMA-ES etc. Results are motivating as compare to earlier traditional approaches. BBO has already been established as one of the most promising recent continuous optimizer. To the best of authors' knowledge, BBO has yet not been applied for the solution of WFLOP. Therefore, it is significant to explore the application of BBO in solving WFLOP.
In a given wind farm, if N turbines are to be placed, then any arrangement of these N turbines in a two-dimensional wind farm represent a potential solution in BBO. Thus j^{th} solution x_j is represented by $x_j = (x_j^1, y_j^1, x_j^2, y_j^2, ..., x_j^N, y_j^N)$. Clearly, the number of decision variables in the problem are $2N$. Where (x_j^t, y_j^t) for $1 \leq t \leq N$ is the position of t^{th} turbine. BBO operators are then applied to a population of such potential solutions to modify so that total energy output is maximum. The implementations of BBO to solve WFLOP is given in Algorithm 2.

Algorithm 2 BBO algorithm for WFLOP

Population size ← N_p;
Generation Index ← $GenIndex$;
Maximum number of generations ← $MaxGen$;
Number of turbine ← N;
Dimension ← $n = 2N$;
Immigration rate ← λ_{im}
Emigration rate ← μ_{em}
Initialize the solution $(x_j^1, y_j^1, x_j^2, y_j^2, ..., x_j^N, y_j^N)$ $j \leq N_p$;
Compute vel_def_i, c_i, $E(P_i)$ for $i = 1, ..., N$ and $\sum_{i=1}^{N} E(P_i)$ for each solution (habitat);
for $GenIndex = 1$ to $MaxGen$ **do**
  According to the value of λ_{im} and μ_{em} Select habitat for migration;
  Apply migration as in Algorithm 1
  Apply mutation as in Algorithm 1
  Re-compute vel_def_i, c_i, $E(P_i)$ for $i = 1, ..., N$ and $\sum_{i=1}^{N} E(P_i)$ for each modified habitat;
  Stop, if termination criterion satisfied
end for

In this article, we considered two types of wind data sets (wind data set (I) given in Table 1, wind data set (II) given in Table 2). Each wind data set is distributed in 24 small parts of 0 to 23 (denoted by $l-1$). In Tables 1 and 2, wind data set is distributed in 24 intervals of wind direction (15^0 in each interval). In each wind direction, interval (from θ_{l-1} to θ_l), Weibull distribution shape parameter (k), Weibull distribution scale parameter (c) and wind blowing probability (ω_{l-1}) are given in Tables 1 and 2. BBO is applied with these wind data sets (Table 1, Table 2) to solve WFLOP. Considered radii of the wind farms are 500 (m), 750 (m) and 1000 (m). The number of feasible turbines varies from 2 to maximum 15 in the several wind farms. In the next section, results of WFLOP using BBO are reported and compared with other state of the art algorithms.

Table 1: Wind data set (I)

l-1	θ_{l-1}	θ_l	k	c	ω_{l-1}
0	0	15	2	13	0
1	15	30	2	13	0.01
2	30	45	2	13	0.01
3	45	60	2	13	0.01
4	60	75	2	13	0.01
5	75	90	2	13	0.2
6	90	105	2	13	0.6
7	105	120	2	13	0.01
8	120	135	2	13	0.01
9	135	150	2	13	0.01
10	150	165	2	13	0.01
11	165	180	2	13	0.01
12	180	195	2	13	0.01
13	195	210	2	13	0.01
14	210	225	2	13	0.01
15	225	240	2	13	0.01
16	240	255	2	13	0.01
17	255	270	2	13	0.01
18	270	285	2	13	0.01
19	285	300	2	13	0.01
20	300	315	2	13	0.01
21	315	330	2	13	0.01
22	330	345	2	13	0.01
23	345	360	2	13	0

5. Results and discussion

The following experimental setting is adopted to see the performance of BBO.
- Population size, $N_p = 50, 100$
- Mutation probability $= 0.01$
- Elitism size $= 2$
- Maximum immigration rate $= 1$

Table 2: Wind data set (II)

l-1	θ_{l-1}	θ_l	k	c	ω_{l-1}
0	0	15	2	7	0.0002
1	15	30	2	5	0.008
2	30	45	2	5	0.0227
3	45	60	2	5	0.0242
4	60	75	2	5	0.0225
5	75	90	2	4	0.0339
6	90	105	2	5	0.0423
7	105	120	2	6	0.029
8	120	135	2	7	0.0617
9	135	150	2	7	0.0813
10	150	165	2	8	0.0994
11	165	180	2	9.5	0.1394
12	180	195	2	10	0.1839
13	195	210	2	8.5	0.1115
14	210	225	2	8.5	0.0765
15	225	240	2	6.5	0.008
16	240	255	2	4.6	0.0051
17	255	270	2	2.6	0.0019
18	270	285	2	8	0.0012
19	285	300	2	5	0.001
20	300	315	2	6.4	0.0017
21	315	330	2	5.2	0.0031
22	330	345	2	4.5	0.0097
23	345	360	2	3.9	0.0317

- Maximum emigration rate = 1
- Maximum number of generations/iterations = 50
- Total number of runs/simulations = 10

Values of other parameters used in optimization model (8) with (10) are as below:

- Rotor radius, $R = 38.5$ (m)
- Wind cut-in speed, $v_{cut-in} = 3.5$ (m/s)
- Wind rated speed, $v_{rated} = 14$ (m/s)
- Rated power for wind speed, $P_{rated} = 1500$ (kW)

The parameter used in linear power curve function, $\lambda' = 140.86$, $\eta = -500$. The thrust coefficient c_T is acceded to be 0.8 and the spreading constant K is acceded to be 0.075. Wind speed is divided into $N_v = 20$ intervals of 0.5 (m) each, where the initial point is v_{cut-in} and final point is v_{rated}. Similarly, the wind direction is divided into $N_\theta = 23$ intervals of 15^0 each.

For the wind data set (I), given in Table 1 the Weibull parameters (k=2 and c=13) are fixed in each interval of wind direction. But the wind blowing probability varies in several wind directions. Wind blowing probability in initial wind direction (from 0^0 to 15^0) and in last wind direction (from 345^0 to 360^0) is 0 ($\omega_0 = 0$ and $\omega_{23} = 0$). The wind blowing probability is the highest in the wind direction interval from 75^0 to 105^0. In the wind direction interval from 75^0 to 90^0, the wind blowing probability is 0.2 ($\omega_5 = 0.2$) and in the wind direction interval from 90^0 to 105^0, the wind blowing probability is 0.6 (ω_6=0.6). The wind blowing probability in other wind direction is 0.01 (ω_l=0.01, $l \neq 0$, 5, 6, 23).

For the wind data set (II) given in Table 2, shape parameter is fixed ($k = 2$) but Weibull distribution scale parameter (c) is not fixed in each interval of the wind direction. The lowest value of scale parameter (c) is 3, in the wind direction interval (from 255^0 to 70^0) and the highest value of c is 10 in two wind direction intervals (from 165^0 to 180^0 and 180^0 to 195^0).

Since the power production is directly proportional to the number of wind turbines. Therefore, for a given wind farms sizes, we wish to find out maximum limit of the feasible wind turbines and their placements with minimum wake loss or maximum power output. The results using BBO for considered farm radii (500 (m), 750 (m) and 1000 (m)) are discussed below.

Wind farm radius 500 (m): Tables 3 and 4, illustrate the expected power with wake loss in the farm radius 500 (m) for the wind data set (I) and the wind data set (II), respectively. In Tables 3 and 4, columns 1 and 2 report the number of turbines and the ideal expected power corresponding to the number of turbines. Results of wind data sets (I) and (II) from earlier studies are reported in column 3 - 7. Evolutionary algorithm (EA) results (column 3), Ant colony optimization (ACO) (columns 4 and 5) and Particle filtering (PF) approach results (columns 6 and 7) are reproduced from [13], [16] and [17], respectively. In Tables 3 and 4, columns 8 and 9 report the best-expected power with wake loss for 50 population size (50 N_p) and 100 population size (100 N_p) corresponding to the number of turbines using BBO algorithm. Finally in Tables 3 and 4, columns 10 and 11 report the average of expected power with wake loss in 10 runs for 50 population size (50 N_p) and 100 population size (100 N_p) corresponding to the number of turbines using BBO algorithm. Previously, many approaches have been applied to calculate maximum expected power production of wind turbines. In [13], Andrew and Kusiak solved this problem by the evolutionary algorithm (EA) and able to find the optimal placement of 6 turbines on farm radius 500 (m). Authors were in the view that there is no more optimal space for more than 6 turbines in the same farm area for both wind data sets (wind data set (I) and wind data set (II)). Then in [16], Eroğlu and Seçkiner developed the efficient solution by ant colony optimization (ACO) and succeeded to find the optimal location of maximum 8 number of turbines on the same wind farm. In ACO, the ideal expected power is compared with ACO (best) and ACO (average of 10-run). From the Tables 3 and 4, it is clear that up to 3 turbines, best-expected power output is equal to the ideal expected power output. Again in [17], Eroğlu and Seçkiner improved the optimal position of wind turbines using particle filtering (PF) approach. From the comparison data given in Tables 3 and 4, PF approach is better than EA and ACO algorithm. But still the best-expected power is not equal to the ideal expected power (optimum) for more than 3 turbines. These two solutions motivate authors to explore the performance of BBO to WFLOP. It is expected that BBO could provide the optimal placement of more turbines on the same farm. Tables 3 and 4 show the comparison of expected power and wake loss developed by EA, ACO, PF and BBO algorithm. Table 3 shows that up to 7 turbines, the best-expected power is equal to the ideal expected power but 8 turbines can be placed with the negligible amount of wake loss. But Table 4 shows that up to 6 number of turbines, the best-expected power is equal to the ideal expected power but 7 and 8 turbines can be placed with the negligible amount of wake loss.

In this way, we can not place more than 8 turbines on the wind farm of radius 500 (m). Therefore expected power generation capacity of wind farm of radius 500 (m) using BBO algorithm is better than EA, ACO and PF approach.

Figures 5 and 6, illustrate the optimum location of wind turbines from 2 to 8 turbines for the wind data set (I) and the wind data set (II), respectively. Here in figures 5(a)-5(g) and 6(a)- 6(g), turbines' best position is seen for the best-expected power (column 8) given in Tables 3 and 4, respectively.

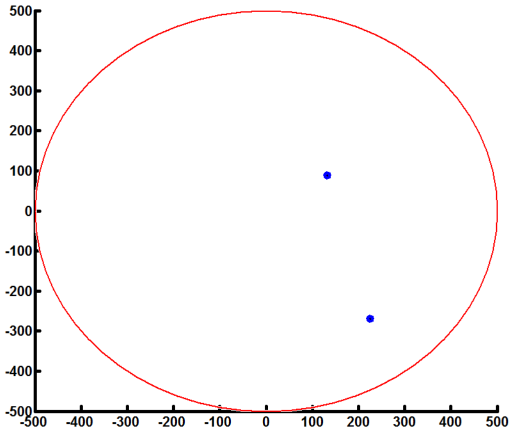
The change in wake loss with respect to iterations can be observed using fitness curve in figures 7 and 8. Fitness curve for wind data sets (I) and (II) with population size 50 are given in figures 7(a) and 7(b), respectively. Similarly the fitness curve with population size 100 are given in figures 8(a) and 8(b), respectively. Only for feasible fitness values, fitness curves are shown. It can be easily observed that within 50 iterations, the fitness curve becomes parallel to the horizontal axis, i.e. the chances of further improvement are negligible. Thus 50 iterations seems to be sufficient for experiments. For 2, 3 and 4 turbines cases, the optimum solution is reached in early iterations and therefore corresponding fitness curves are not shown here.

Table 3: Expected power and wake loss (*in kW*) in the wind farm of radius 500 (*m*) for the wind data set (I)

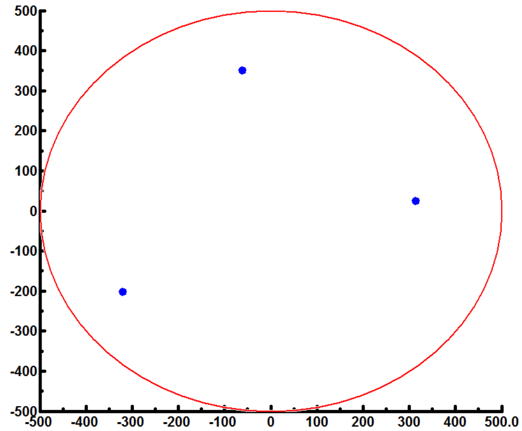
Number of turbines	Ideal	EA /Wake Loss	ACO (best) /Wake Loss	ACO (average of 10-run) /Wake Loss	PF (best) /Wake Loss	PF (average of 10-run) /Wake Loss	BBO (best) /Wake Loss		BBO (average of 10-run) /Wake Loss	
							50 N_p	100 N_p	50 N_p	100 N_p
2	28091.47	28083.42 /8.05	28091.47 /0	28091.47 /0	28091.47 /0	28091.47 /0	28091.47 / 0	28091.47 /0	28091.47 /0	28091.5 /0
3	42137.21	42101.06 /36.15	42137.21 /0	42130.87 /6.34	42137.21 /0	42128.32 /8.89	42137.21 / 0	42137.21 /0	42137.21 /0	42137.2 /0
4	56182.95	56057.77 /125.18	56150.13 /32.82	56128 /54.95	56152.58 /30.37	56135.28 /47.67	56182.95 / 0	56182.95 /0	56167.85 /15.1	56171.1 /11.89
5	70228.69	69922.97 /305.72	70113.48 /115.20	70086.29 /142.40	70122.64 /106.05	70085.66 /143.03	70228.69 / 0	70228.69 /0	70196.42 /32.27	70207.2 /21.53
6	84274.42	83758.79 /515.63	84042.34 /231.09	84006.37 /268.05	84047.05 /227.37	84007.84 /266.58	84274.42 / 0	84274.42 / 0	84236.19 /38.23	84167.4 /106.98
7	98320.16	—— ——	97905.99 /414.17	97822.66 /497.50	97918.69 /401.47	97869.73 /450.43	98320.16 / 0	98320.16 / 0	98265.71 /54.45	98280.5 /239.71
8	112365.9	—— ——	111589.7 /776.20	111414.82 /951.08	111694.24 /671.66	111498.83 /867.07	112318.78 / 47.12	112322.9 /43.01	112168.34 /197.56	111894 /471.99
9	126411.64	——	——	——	——	——	Infeasible	Infeasible	Infeasible	Infeasible

Table 4: Expected power and wake loss (*in kW*) in the wind farm of radius 500 (*m*) for the wind data set (II)

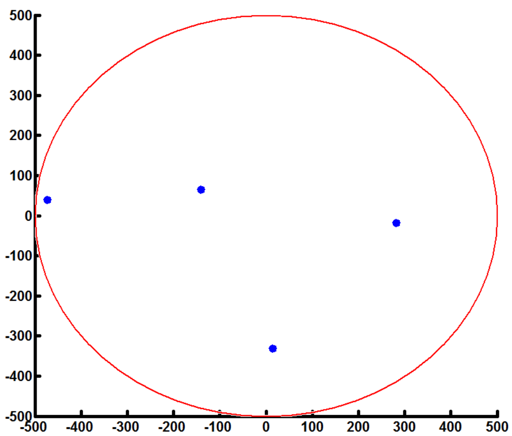
Number of turbines	Ideal	EA /Wake Loss	ACO (best) /Wake Loss	ACO (average of 10-run) /Wake Loss	PF (best) /Wake Loss	PF (average of 10-run) /Wake Loss	BBO (best) /Wake Loss		BBO (average of 10-run) /Wake Loss	
							50 N_p	100 N_p	50 N_p	100 N_p
2	14631.37	14631.21 /0.16	14631.37 /0	14631.37 /0	14631.37 /0	14631.37 /0	14631.37 /0	14631.37 /0	14631.37 /0	14631.37 /0
3	21947.06	21925.16 /21.90	21947.06 /0	21928.07 /18.99	21947.06 /0	21915.78 /31.28	21947.06 /0	21947.06 /0	21947.06 /0	21947.06 /0
4	29262.75	29,113.71 /149.04	29,204.65 /58.10	29,174.20 /88.55	29217.83 /44.92	29182.38 /80.37	29262.75 /0	29262.75 /0	29232.54 /30.21	29232.74 /30.01
5	36578.44	36,316.23 /262.21	36,389.27 /189.16	36,256.10 /322.34	36421.55 /156.89	36284.07 /294.37	36578.44 /0	36578.44 /0	36507.35 /71.09	36508.67 /69.77
6	43894.12	43,195.84 /698.28	43,202.50 /691.62	43,125.19 /768.93	43326.88 /567.24	43181.70 /712.42	43894.12 /0	43894.12 /0	43795.59 /98.53	43697.25 /196.87
7	51209.81	——	49,943.97 /1265.84	49,763.76 /1446.06	50011.33 /1198.48	49819.71 /1390.10	51208.05 /1.76	51209.81 /0	50993.02 /216.79	50914.72 /295.09
8	58525.50	——	56,453.73 /2071.77	56,316.15 /2209.35	56664.57 /1860.93	56498.03 /2027.47	58401.71 /123.79	58468.88 /56.62	58128.58 /396.92	58020.39 /505.11
9	65841.19	——	——	——	——	——	Infeasible	Infeasible	Infeasible	Infeasible



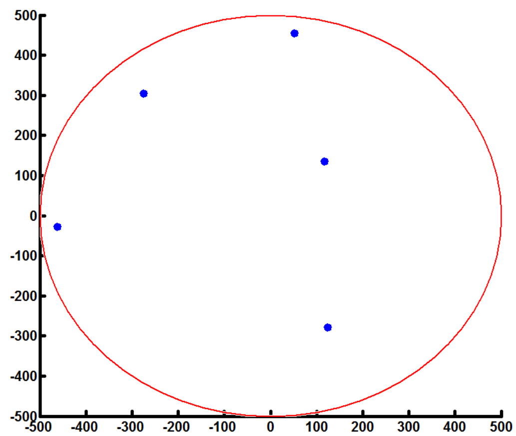
(a) 2 turbine



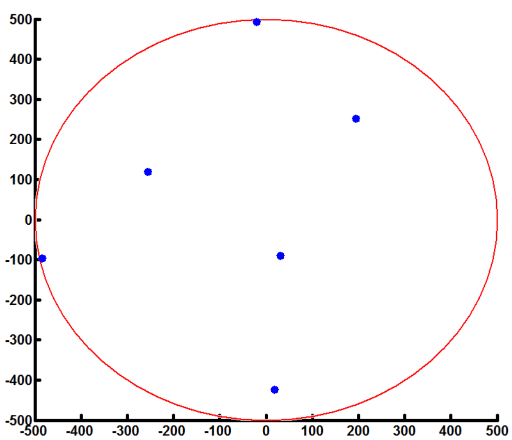
(b) 3 turbine



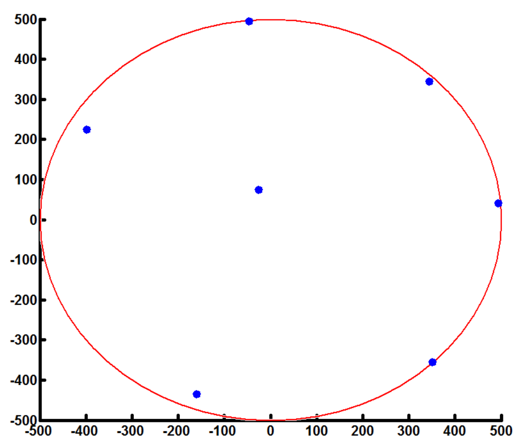
(c) 4 turbine



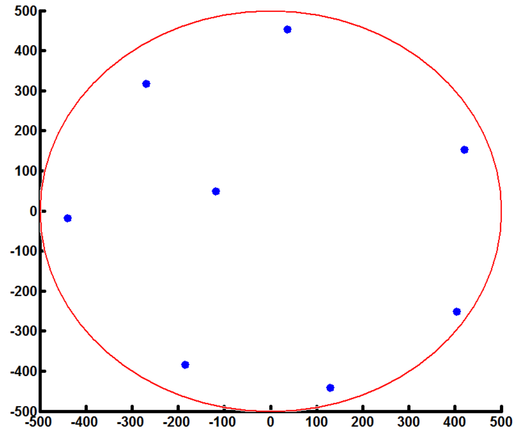
(d) 5 turbine



(e) 6 turbine

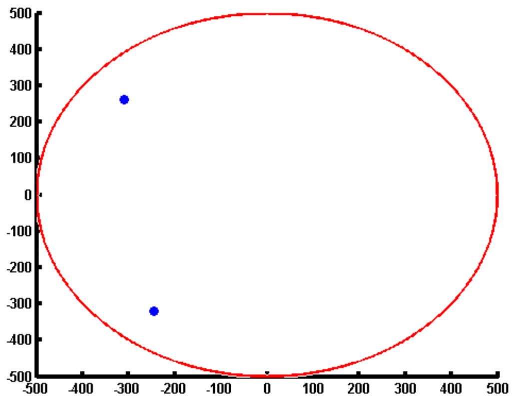


(f) 7 turbine

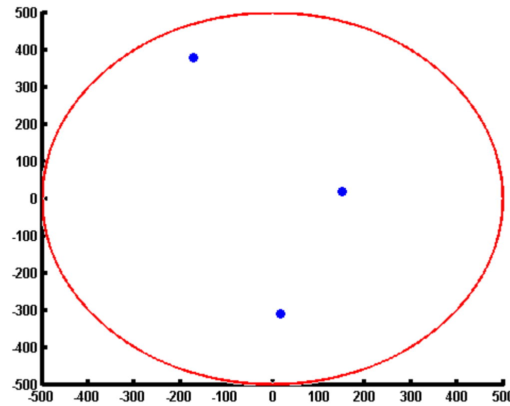


(g) 8 turbine

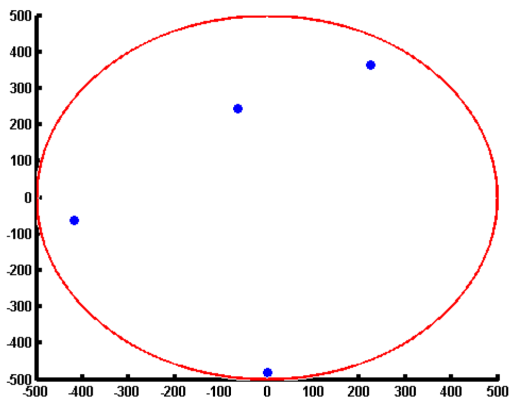
Figure 5: Turbines' location in the wind farm of radius 500 (m) for the wind data set (I)



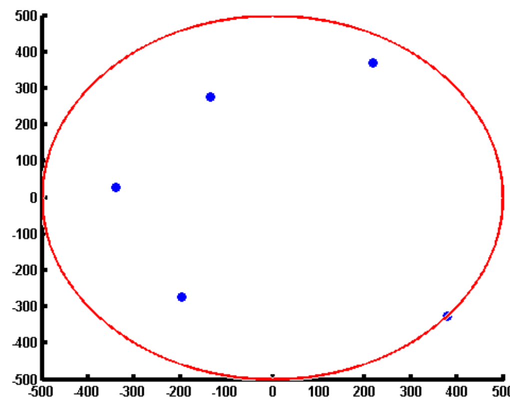
(a) 2 turbine



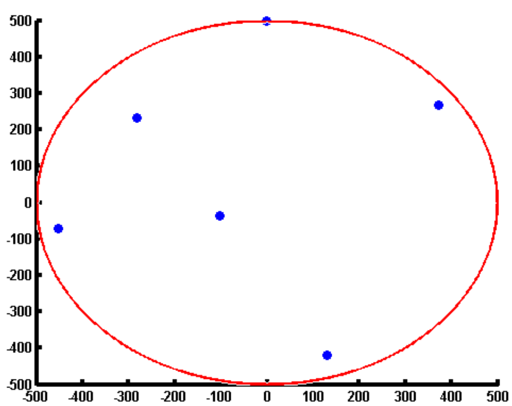
(b) 3 turbine



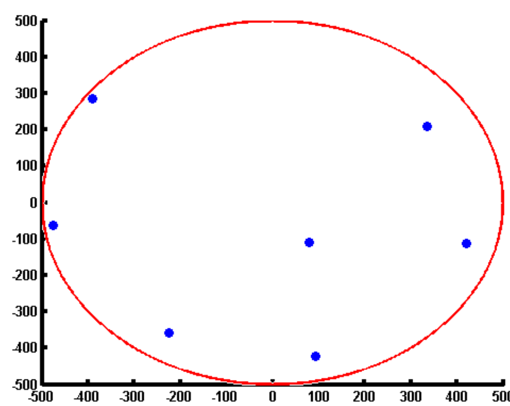
(c) 4 turbine



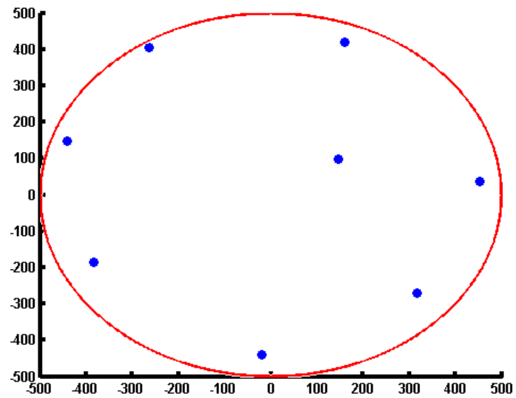
(d) 5 turbine



(e) 6 turbine

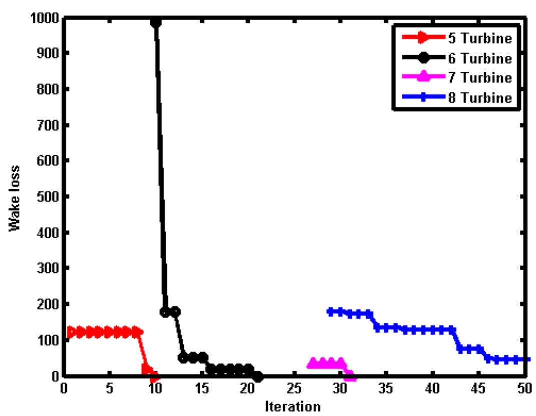


(f) 7 turbine

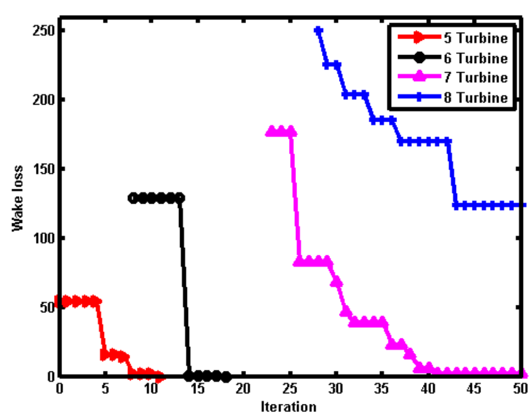


(g) 8 turbine

Figure 6: Turbines' location in the wind farm of radius 500 (m) for the wind data set (II)

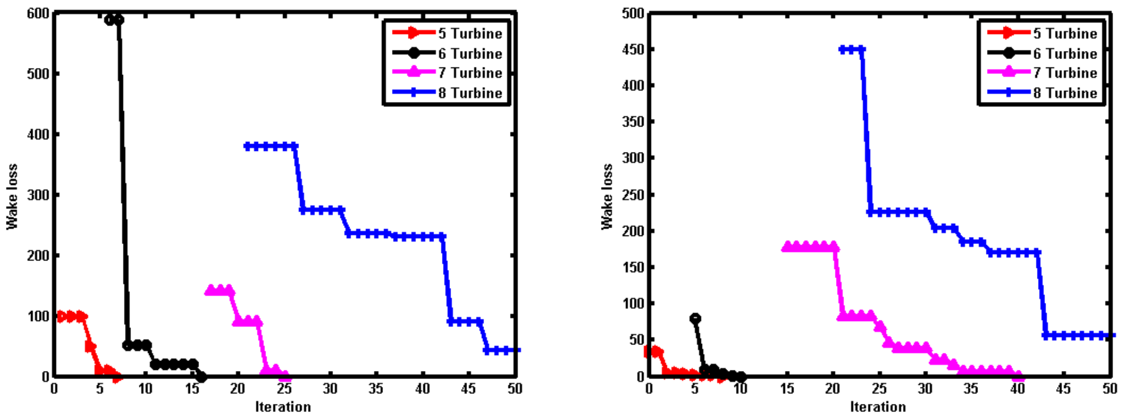


(a) Wake loss vs iteration for the wind data set (I)



(b) Wake loss vs iteration for the wind data set (II)

Figure 7: Fitness curve for the wind farm of radius 500 (m) with $N_p = 50$



(a) Wake loss vs iteration for the wind data set (I)

(b) Wake loss vs iteration for the wind data set (II)

Figure 8: Fitness curve for the wind farm of radius 500 (m) with $N_p = 100$

Wind farm radius 750 (m): We succeeded to find the optimal placement of only up to 7 turbines for the wind data set (I) and up to 6 turbines for the wind data set (II) in the wind farm of radius 500 (m) without any wake loss. So we require more space to establish the wind turbines without any wake loss. In this way searching the possibility of the optimal placement of more turbines, farm radius is increased by 250 (m). Now further searching for the optimal placement of more number of turbines and also find the maximum possible limit of the feasible wind turbines on the wind farm of radius 750 (m). In Tables 5 and 6, columns 1 and 2 report the number of turbines and the ideal expected power corresponding to the number of turbines. In Tables 5 and 6, columns 3 and 4 report the best-expected power with wake loss for 50 population size (50 N_p) and 100 population size (100 N_p) corresponding to the number of turbines using BBO algorithm. Finally in Tables 5 and 6, columns 5 and 6 report the average of expected power with wake loss in 10 runs for 50 population size (50 N_p) and 100 population size (100 N_p) corresponding to the number of turbines using BBO algorithm. Table 5 illustrate the best-expected power, expected power of average of 10-run and wake loss up to 12 number of turbines for the wind data set (I) and data is compared with only the ideal expected power (optimum) because no data available in the literature for this wind farm size. In the wind farm of radius 500 (m), wake loss is measured if 8 number of turbines establish but no wake loss is found in the wind farm of radius 750 (m) up to 9 number of turbines for the wind data set (I). As well as Table 6 illustrates the best-expected power, expected power of average of 10-run and wake loss up to 12 turbines for the wind data set (II) and data is compared with only the ideal expected power (optimum) because no data available in the literature for this wind farm size. In the wind farm of radius 500 (m), wake loss is measured if 7 number of turbines establish but no wake loss is found in the wind farm of radius 750 (m) up to 8 number of turbines for the wind data set (II). In the wind farm of radius 750 (m) only up to 12 number of turbines can be established. Tables 5 and 6 clear the maximum limit of the feasible number of turbines for specific wind farm radius 750 (m) can not be exceeded by 12 number of turbines.

Fig. 9, illustrates the optimum location of wind turbines from 8 to 12 turbines. Here in figures 9(a)-9(e), turbines' best position is seen for the best-expected power (column 3) given in Table 5. Again Fig. 10, illustrates the optimum location of wind turbines from 7 to 12 number of turbines. Here in figures 10(a)-10(f), turbines' best position is seen for the best-expected power (column 3) given in Table 6.

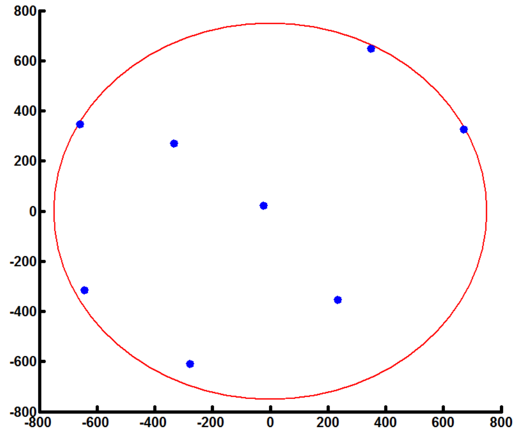
Similar to 500 (m) farm case, fitness curves are shown in figures 11 and 12.

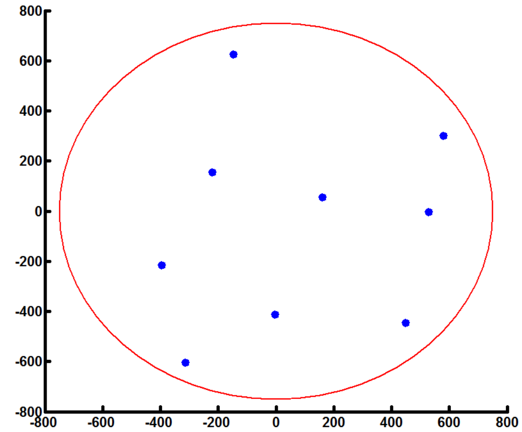
Table 5: Expected power and wake loss (*in kW*) in the wind farm of radius 750 (m) for the wind data set (I)

Number of turbines	Ideal	BBO (best) /Wake Loss		BBO (average of 10-run) /Wake Loss	
		50 N_p	100 N_p	50 N_p	100 N_p
2	28091.47	28091.47 /0	28091.47 /0	28091.47 /0	28091.47 /0
3	42137.21	42137.21 /0	42137.21 /0	42137.21 /0	42137.21 /0
4	56182.95	56182.95 /0	56182.95 /0	56182.95 /0	56182.95 /0
5	70228.69	70228.69 /0	70228.69 /0	70228.69 /0	70228.69 /0
6	84274.42	84274.42 /0	84274.42 /0	84261.58 /12.84	84245.1 /29.32
7	98320.16	98320.16 /0	98320.16 /0	98298.66 /21.5	98280.17 /39.99
8	112365.9	112365.9 /0	112365.9 /0	112319.27 /46.63	112306.32 /59.58
9	126411.64	126411.64 /0	126411.64 /0	126313.25 /98.39	126302.42 /109.22
10	140457.37	140441.79 /15.58	140447.3 /10.07	140303.54 /153.46	140217.94 /239.43
11	154503.11	154417.5 /85.61	154434.14 /68.97	154111.64 /391.47	154128.72 /374.39
12	168548.85	168449.56 /99.29	168474.97 /73.88	167967.46 /581.39	167901.04 /647.81
13	182594.59	Infeasible	Infeasible	Infeasible	Infeasible

Table 6: Expected power and wake loss (*in kW*) in the wind farm of radius 750 (*m*) for the wind data set (II)

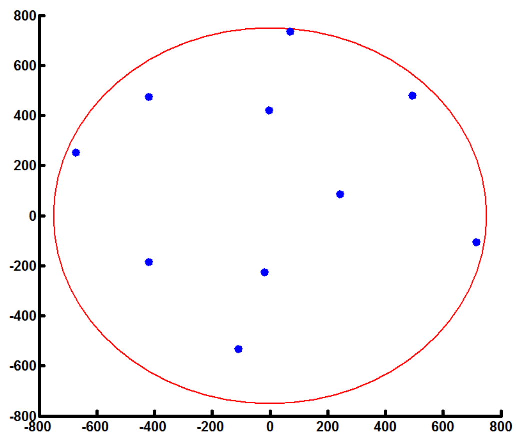
Number of turbines	Ideal	BBO (best) /Wake Loss		BBO (average of 10-run) /Wake Loss	
		50 N_p	100 N_p	50 N_p	100 N_p
2	14631.37	14631.37 /0	14631.37 /0	14631.37 /0	14631.37 /0
3	21947.06	21947.06 /0	21947.06 /0	21947.06 /0	21947.06 /0
4	29262.75	29262.75 /0	29262.75 /0	29262.75 /0	29262.75 /0
5	36578.44	36578.44 /0	36578.44 /0	36578.44 /0	36578.44 /0
6	43894.12	43894.12 /0	43894.12 /0	43877.4 /16.72	43852.33 /41.79
7	51209.81	51209.81 /0	51209.81 /0	51164.73 /45.08	51154.82 /54.99
8	58525.50	58525.50 /0	58525.50 /0	58435.71 /89.79	58428.48 /97.02
9	65841.19	65841.05 /0.14	65841.19 /0	65764.34 /76.85	65742.68 /98.51
10	73156.87	73156.67 / 0.20	73156.87 /0	73044 /112.87	72942.28 /214.59
11	80472.56	80456.48 /16.08	80457.35 /15.21	80216.84 /255.72	80090.17 /382.39
12	87788.25	87736.94 /51.31	87738.34 /49.91	87486.39 /301.86	87276.02 /512.23
13	95103.94	Infeasible	Infeasible	Infeasible	Infeasible

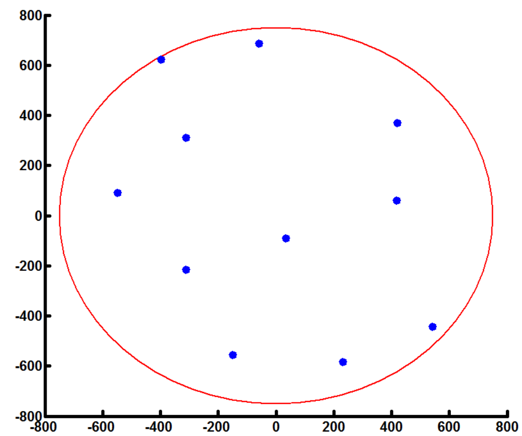




(a) 8 turbine

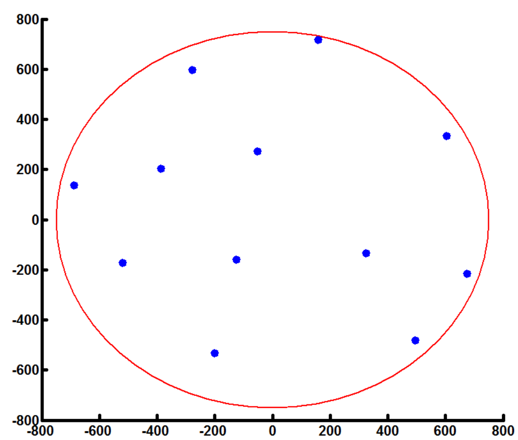
(b) 9 turbine





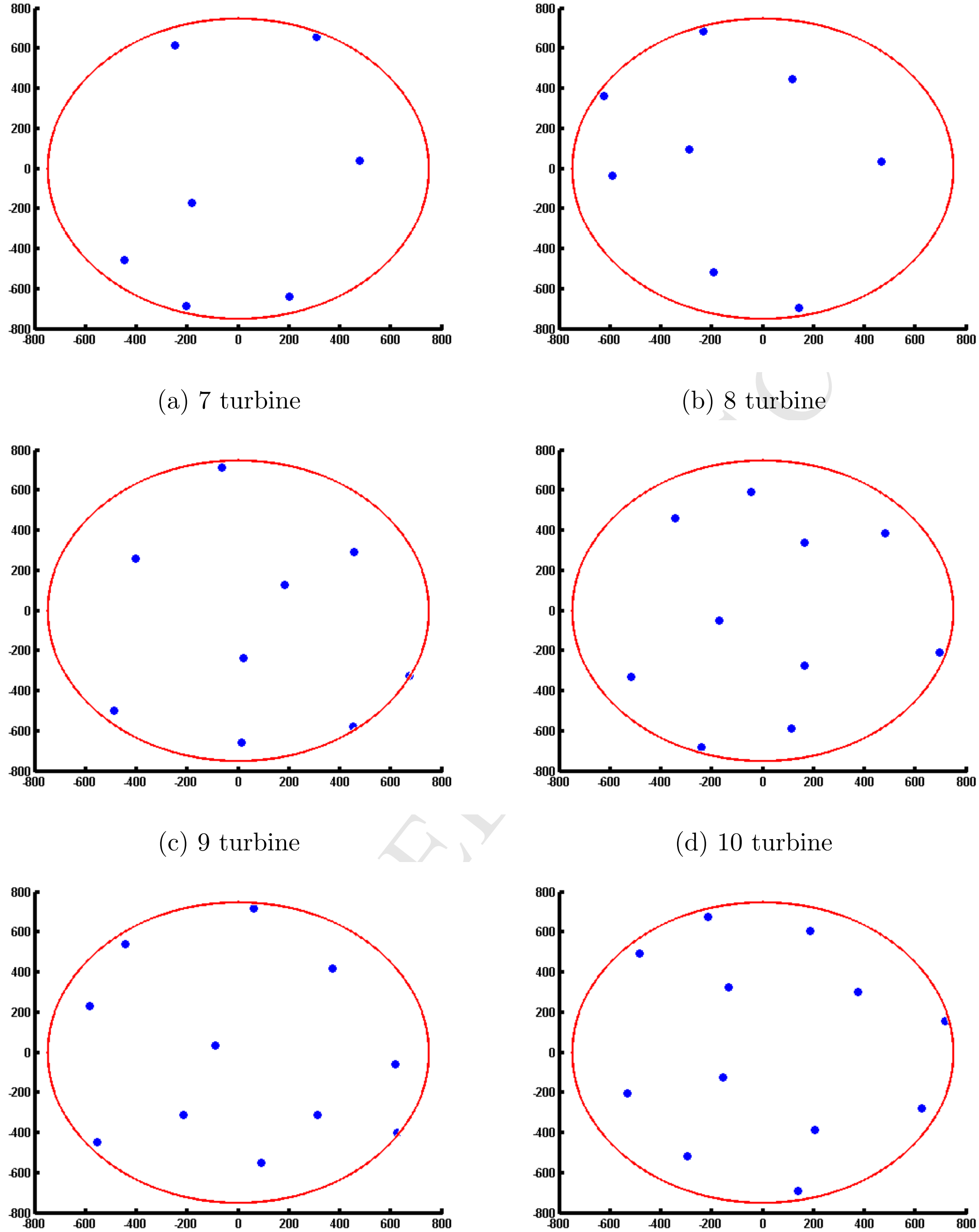
(c) 10 turbine

(d) 11 turbine



(e) 12 turbine

Figure 9: Turbines' location in the wind farm of radius 750 (m) for the wind data set (I)



(a) 7 turbine

(b) 8 turbine

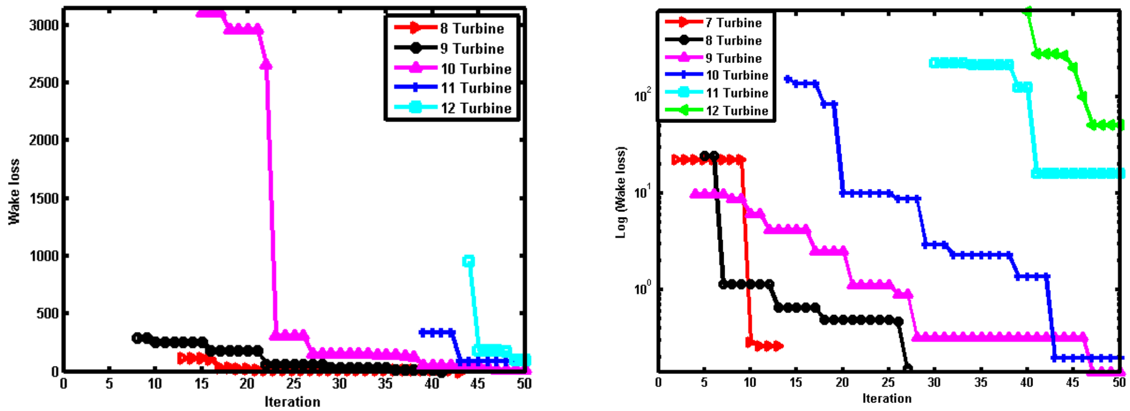
(c) 9 turbine

(d) 10 turbine

(e) 11 turbine

(f) 12 turbine

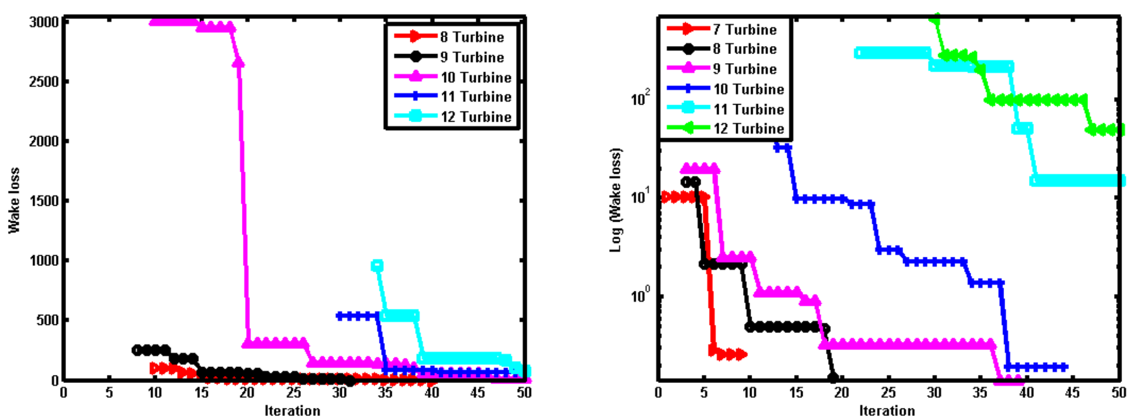
Figure 10: Turbines' location in the wind farm of radius 750 (m) for the wind data set (II)



(a) Wake loss vs iteration for the wind data set (I)

(b) Wake loss vs iteration for the wind data set (II)

Figure 11: Fitness curve for the wind farm of radius 750 (m) with $N_p = 50$



(a) Wake loss vs iteration for the wind data set (I)

(b) Wake loss vs iteration for the wind data set (II)

Figure 12: Fitness curve for the wind farm of radius 750 (m) with $N_p = 100$

Wind farm radius 1000 (m): The same challenge is arises if we increase the farm radius. Again we need to search the maximum possible limit of the feasible number of turbines for the best-expected power in the wind farm of radius 1000 (m). In Tables 7 and 8, columns 1 and 2 report the number of turbines and the ideal expected power corresponding to the number of turbines. In Tables 7 and 8, columns 3 and 4 report the best-expected power with wake loss for 50 population size (50 N_p) and 100 population size (100 N_p) corresponding to the number of turbines using BBO algorithm. Finally in Tables 7 and 8, columns 5 and 6 report the average of expected power with wake loss in 10 runs for 50 population size (50 N_p) and 100 population size (100 N_p) corresponding to the number of turbines by BBO algorithm. Tables 7 and 8 illustrate the best-expected power, expected power of average of 10-run and wake losses up to 15 number of turbines for the wind data set (I) and the wind data set (II), respectively. The developed data is compared with the ideal expected power (optimum) given in Tables 7 and 8. Tables 7 and 8 illustrate the best-expected power and wake loss for exceeded number of turbines. Here in this wind farm the best-expected power is same as the ideal expected power until 11 number of turbines and the wind farm capacity is increased up to 15 number of turbines given in Tables 7 and 8. Therefore, in this wind farm number of the feasible turbines can not be exceeded by 15. So from the above study and experiments, BBO algorithm is able to declare the maximum limit of the feasible wind turbines on the selected wind farm.

Fig. 13, illustrates the optimum location of wind turbines from 10 to 15 turbines. Here in figures 13(a)-13(f), turbines' best position is seen for the best-expected power (column 3) given in Table 7. Again Fig. 14, illustrates the optimum location of wind turbines from 9 to 15 number of turbines. Here in figures 14(a)-14(g), turbines' best position is seen for the best-expected power (column 3) given in Table 8.

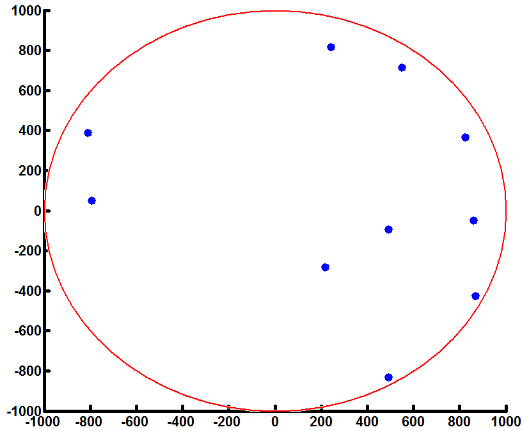
Similar to 500 (m) and 750 (m) farm cases, fitness curves are shown in figures 15 and 16.

Table 7: Expected power and wake loss (*in kW*) in the wind farm of radius 1000 (*m*) for the wind data set (I)

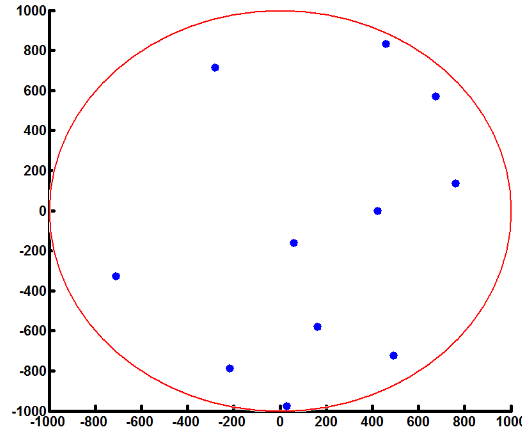
Number of turbines	Ideal	BBO (best) /Wake Loss		BBO (average of 10-run) /Wake Loss	
		50 N_p	100 N_p	50 N_p	100 N_p
2	28091.47	28091.47 /0	28091.47 /0	28091.47 /0	28091.47 /0
3	42137.21	42137.21 /0	42137.21 /0	42137.21 /0	42137.21 /0
4	56182.95	56182.95 /0	56182.95 /0	56182.95 /0	56182.95 /0
5	70228.69	70228.69 /0	70228.69 /0	70228.69 /0	70228.69 /0
6	84274.42	84274.42 /0	84274.42 /0	84274.42 /0	84274.42 /0
7	98320.16	98320.16 /0	98320.16 /0	98320.16 /0	98320.16 /0
8	112365.9	, 112365.9 /0	, 112365.9 /0	112347.31 /18.59	112338.35 /27.55
9	126411.64	126411.64 /0	126411.64 /0	126359.91 /51.73	126333.03 /78.61
10	140457.37	140457.37 /0	140457.37 /0	140369.66 /87.71	140354.84 /102.53
11	154503.11	154503.11 /0	154503.11 /0	154403.31 /99.8	154379.54 /123.57
12	168548.85	168541.16 /7.69	168547.77 /1.08	168380.92 /167.93	168355.41 /193.44
13	182594.59	182551.17 /43.42	182552.78 /41.81	182365.8 /228.79	182304.01 /290.58
14	196640.32	196580.96 /59.36	196583.66 /56.66	196308.78 /331.54	196264.54 /375.78
15	210686.06	210634.57 /51.49	210634.57 /51.49	210288.44 /397.62	210237.57 /448.49
16	224731.8	Infeasible	Infeasible	Infeasible	Infeasible

Table 8: Expected power and wake loss (*in kW*) in the wind farm of radius 1000 (*m*) for the wind data set (II)

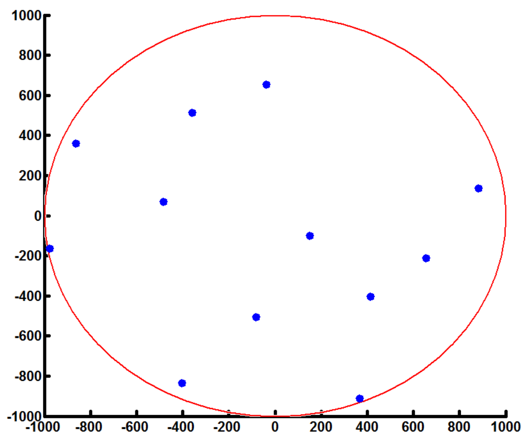
Number of turbines	Ideal	BBO (best) /Wake Loss		BBO (average of 10-run) /Wake Loss	
		50 N_p	100 N_p	50 N_p	100 N_p
2	14631.37	14631.37 /0	14631.37 /0	14631.37 /0	14631.37 /0
3	21947.06	21947.06 /0	21947.06 /0	21947.06 /0	21947.06 /0
4	29262.75	29262.75 /0	29262.75 /0	29262.75 /0	29262.75 /0
5	36578.44	36578.44 /0	36578.44 /0	36578.44 /0	36578.44 /0
6	43894.12	43894.12 /0	43894.12 /0	43894.12 /0	43894.12 /0
7	51209.81	51209.81 /0	51209.81 /0	51190.72 /19.09	51182.14 /27.67
8	58525.50	58525.50 /0	58525.50 /0	58487.53 /37.97	58475.62 /49.88
9	65841.19	65841.19 /0	65841.19 /0	65789.51 /51.68	65762.46 /78.76
10	73156.87	73156.87 /0	73156.87 /0	73065.24 /91.63	73046.09 /110.78
11	80472.56	80472.56 /0	80472.56 /0	80355.47 /117.09	80337.34 /135.22
12	87788.25	87768.71 / 19.54	87769.37 /18.88	87595.63 /192.62	87546.56 /241.69
13	95103.94	94980.22 /123.72	94980.75644 /123.18	94854.4 /249.54	94785.18 /318.76
14	102419.62	102249.93 /169.69	102252.0039 /167.62	102058.08 /361.54	102023.84 /395.78
15	109735.31	109504.22 /231.09	109505.8813 /229.43	109327.72 /407.59	109266.6 /468.71
16	117051	Infeasible	Infeasible	Infeasible	Infeasible



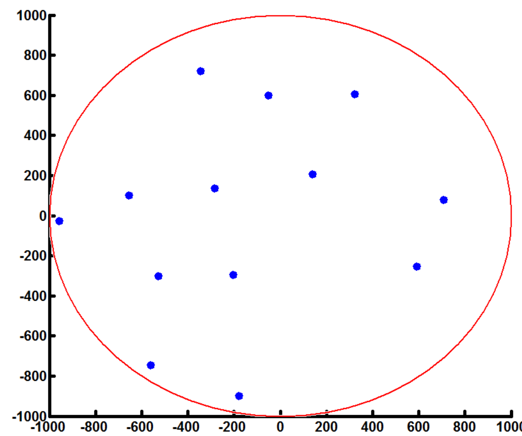
(a) 10 turbine



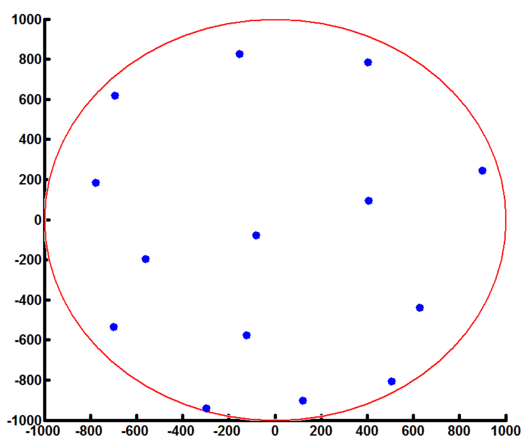
(b) 11 turbine



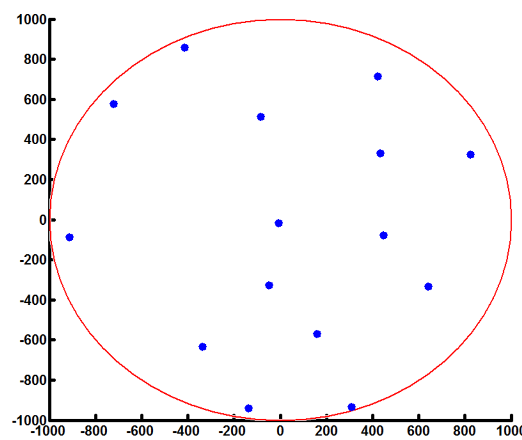
(c) 12 turbine



(d) 13 turbine

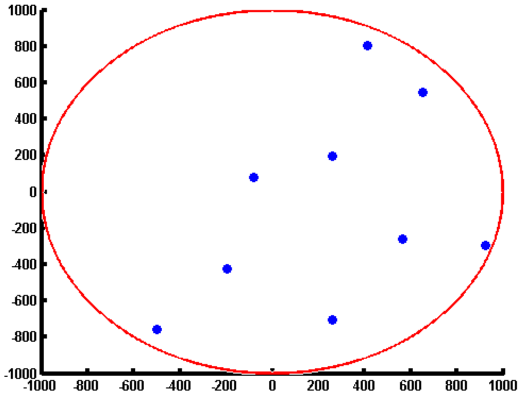


(e) 14 turbine

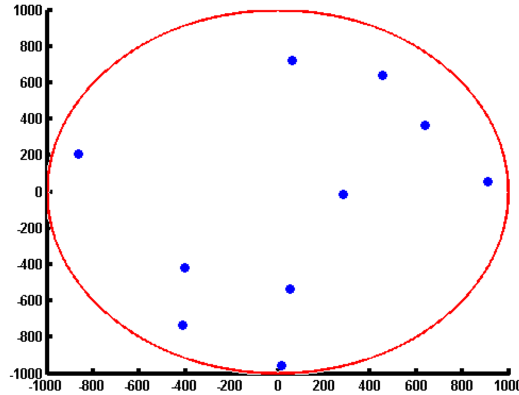


(f) 15 turbine

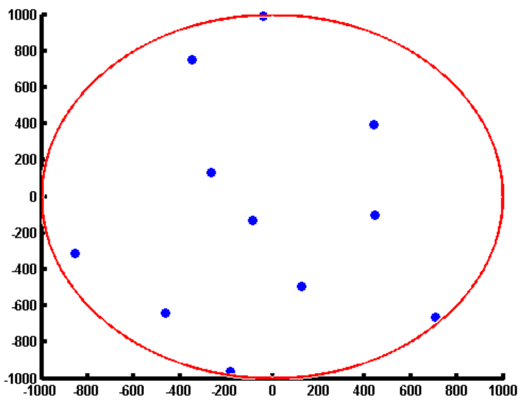
Figure 13: Turbines' location in the wind farm of radius 1000 (m) for the wind data set (I)



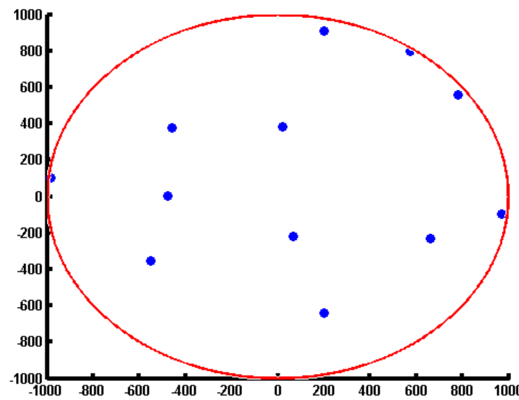
(a) 9 turbine



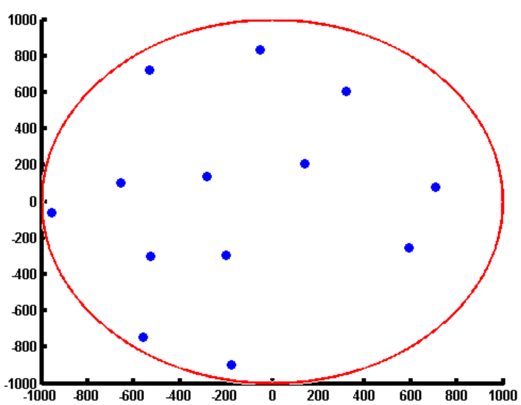
(b) 10 turbine



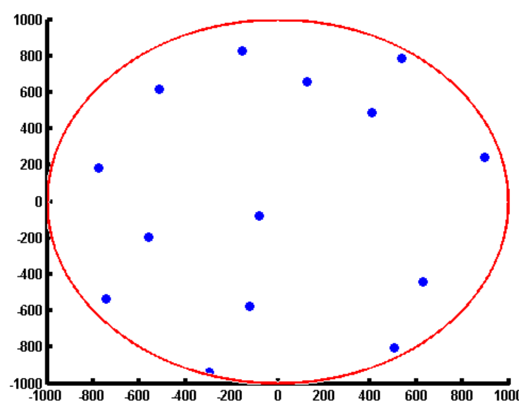
(c) 11 turbine



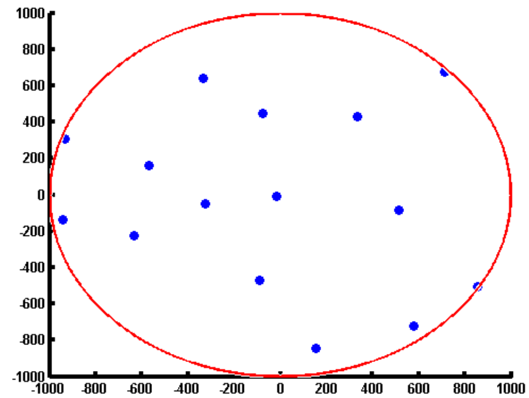
(d) 12 turbine



(e) 13 turbine

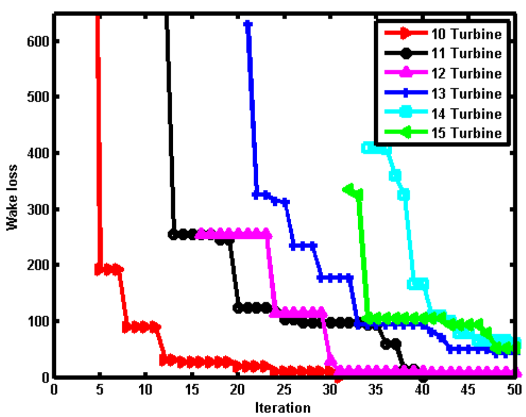


(f) 14 turbine

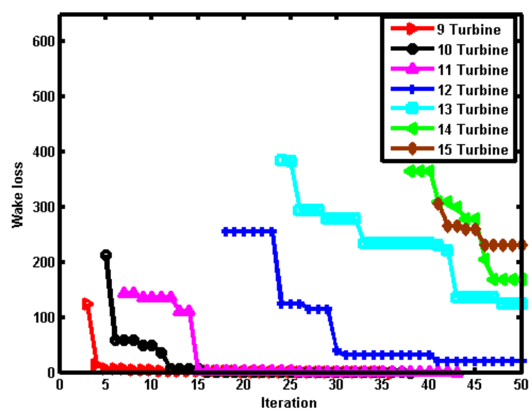


(g) 15 turbine

Figure 14: Turbines' location in the wind farm of radius 1000 (m) for the wind data set (II)

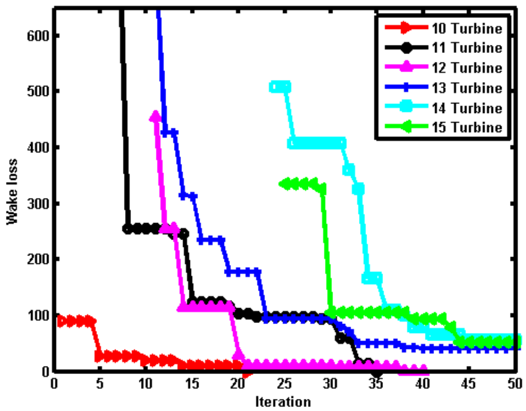


(a) Wake loss vs iteration for the wind data set (I)

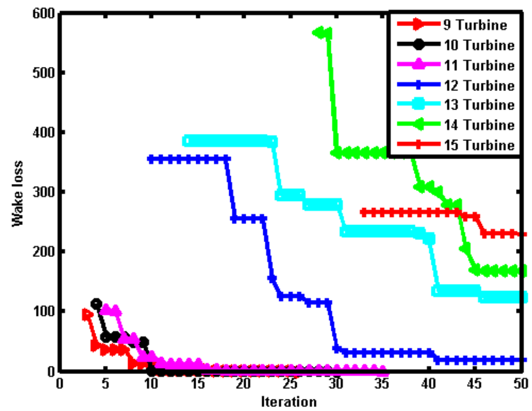


(b) Wake loss vs iteration for the wind data set (II)

Figure 15: Fitness curve for the wind farm of radius 1000 (m) with $N_p = 50$



(a) Wake loss vs iteration for the wind data set (I)



(b) Wake loss vs iteration for the wind data set (II)

Figure 16: Fitness curve for the wind farm of radius 1000 (m) with $N_p = 100$

6. Conclusion

In this paper, WFLOP (wind farm layout optimization problem) is considered to solve using BBO. The main objective of WFLOP is to maximize the energy production along with the reduction of wake effect challenge. In this article, three different circular wind farms with radii 500 (m), 750 (m) and 1000 (m) are considered. The performance of BBO algorithm was evaluated on two wind data sets (wind data set (I) with constant c and wind data set (II) with non-constant c). This paper also recommends the maximum possible number of wind turbines which can be placed in a wind farm without any wake loss. Numerical experiments conclude that BBO is able to find the better optimal placement of wind turbines in the wind farms without any wake loss than prior studies. Earlier methodologies can fit maximum 3 turbines in a farm of radius 500 (m) while BBO can fit maximum 7 turbines in the same farm without any wake loss. Similarly, BBO outperforms for other sizes of wind farms.
Thus BBO is recommended as an efficient solver for WFLOP.

7. Acknowledgment

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Highlights:

- Wind Farm Layout Optimization Problem (WFLOP) is solved using Biogeography-Based Optimization (BBO) method.
- The problem has been dealt in two ways:
    - For a given wind farm and given number of wind turbines, finding the optimal location of wind turbines.
    - Finding the maximum number of wind turbines and their locations, which can be accommodated for a given size of wind farm.
- The experiments have been performed with 500m, 750m and 1000m farm radii and with two different wind data sets having constant and non-constant weibull distribution scale parameter *c*.
- Results have been compared and analyzed with earlier published results.
- The proposed approach has been proved to be competitive for solving WFLOP.